Parametric Bootstrap Tests for Futures Price and Implied Volatility Biases With Application to Rating Dairy Margin Insurance

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            Brian W. Gould, UW-Madison

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U.S. Farm-level Milk Prices, 1961-2014

$/gallon

Milk Price Risk – Market Response

First Dairy Futures
In search of a good design...

Major Reforms
Federal policy creates hedonic milk pricing system (based on protein, butterfat, milk quality, location). No-arbitrage relationship induced between milk, cheese, whey and butter.

Whey and Cheese Futures
6 Active Contracts, 24 months
## Agricultural Product Slate

<table>
<thead>
<tr>
<th>Product Name</th>
<th>Code</th>
<th>Contract</th>
<th>Charts</th>
<th>Last</th>
<th>Change</th>
<th>Open</th>
<th>High</th>
<th>Low</th>
<th>Globex Volume</th>
</tr>
</thead>
<tbody>
<tr>
<td>Class IV Milk</td>
<td>GDKQ4</td>
<td>Aug 2014</td>
<td>OPT</td>
<td>23.49</td>
<td>0</td>
<td>23.70</td>
<td>23.70</td>
<td>23.49</td>
<td>19</td>
</tr>
<tr>
<td>Nonfat Dry Milk</td>
<td>GNFKQ</td>
<td>Aug 2014</td>
<td>OPT</td>
<td>177.30</td>
<td>-0.700</td>
<td>178.250</td>
<td>178.250</td>
<td>177.000</td>
<td>14</td>
</tr>
<tr>
<td>Dry Whey</td>
<td>DYV4</td>
<td>Oct 2014</td>
<td>OPT</td>
<td>60.000</td>
<td>+0.500</td>
<td>59.000</td>
<td>60.225</td>
<td>59.000</td>
<td>10</td>
</tr>
<tr>
<td>Cash-Settled Butter</td>
<td>CBU4</td>
<td>Sep 2014</td>
<td>OPT</td>
<td>231.000</td>
<td>-0.750</td>
<td>236.000</td>
<td>236.000</td>
<td>231.000</td>
<td>22</td>
</tr>
<tr>
<td>Cash-Settled Cheese</td>
<td>CSCU4</td>
<td>Sep 2014</td>
<td>OPT</td>
<td>1.988</td>
<td>-0.002</td>
<td>2.000</td>
<td>2.000</td>
<td>1.988</td>
<td>46</td>
</tr>
</tbody>
</table>
Milk Price Risk – Market Response

Open Interest: All Dairy Futures & Options

Contracts

Year

C₂H₅OH

Percentage of U.S. Corn Crop Consumed by Ethanol Production and Corn Price per Bushel, 1980-2012

- Energy Production Act of 2005 requires increasing levels of ethanol in U.S. gasoline.

Typical Range for U.S. Corn Prices, 1980-2005

Source: Iowa State University Agricultural Marketing Resource Center, Ethanol Usage Projections & Corn Balance Sheet and U.S.D.A. Agricultural Prices
Income over Feed Costs Margins – Squeezed on Both Ends!
Where we use Futures and Options: Interval Forecasts

50% Forecast Interval for Milk-Feed Margin

$ per cwt

## Margin Protection Program – Returns to Participation

### Dairy Program Decision Tool

*Designed by the program on Dairy Markets and Policy*

- **Get Started**
- **Production History**
- **Forecast Margin**
- **Select Coverage**
- **"What if" Game**

#### Coverage Selection & Premium Cost Table

<table>
<thead>
<tr>
<th>Coverage Level</th>
<th>Fees &amp; Premiums</th>
<th>Expected Indemnity</th>
<th>Expected Net Revenue</th>
<th>Expected ROI</th>
</tr>
</thead>
<tbody>
<tr>
<td>$4.00</td>
<td>$100</td>
<td>$0</td>
<td>($100)</td>
<td>(100%)</td>
</tr>
<tr>
<td>$4.50</td>
<td>$1,120</td>
<td>$0</td>
<td>($1,120)</td>
<td>(100%)</td>
</tr>
<tr>
<td>$5.00</td>
<td>$2,310</td>
<td>$0</td>
<td>($2,310)</td>
<td>(100%)</td>
</tr>
<tr>
<td>$5.50</td>
<td>$4,860</td>
<td>$1</td>
<td>($4,859)</td>
<td>(100%)</td>
</tr>
<tr>
<td>$6.00</td>
<td>$7,240</td>
<td>$12</td>
<td>($7,228)</td>
<td>(100%)</td>
</tr>
<tr>
<td>$6.50</td>
<td>$13,020</td>
<td>$43</td>
<td>($12,977)</td>
<td>(100%)</td>
</tr>
<tr>
<td>$7.00</td>
<td>$35,698</td>
<td>$124</td>
<td>($35,574)</td>
<td>(100%)</td>
</tr>
<tr>
<td>$7.50</td>
<td>$46,340</td>
<td>$303</td>
<td>($46,037)</td>
<td>(99%)</td>
</tr>
<tr>
<td>$8.00</td>
<td>$62,490</td>
<td>$652</td>
<td>($61,838)</td>
<td>(99%)</td>
</tr>
</tbody>
</table>

Coverage Percent = 85%
### Livestock Gross Margin – Dairy Cattle

#### LGMDairy.com
Leaders in LGM Dairy Insurance
Contact us to sign up

#### MARKETS
- [LGM Dairy FAQ's](#)
- [Contact Us](#)
- [Basic Margin](#)

#### CONTACT US
- **SALES**
  - LGMDairy@agriskman.com
  - 715-637-Milk (6455)
  - Contact us by webform

<table>
<thead>
<tr>
<th>Month</th>
<th>Mailbox Milk Estimate</th>
<th>Energy (corn) cost est per 100 weight of milk</th>
<th>Protein (meal) cost est per 100 weight of milk</th>
<th>Milk Margin after full feed cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>July-14</td>
<td>$22.64</td>
<td>$3.93</td>
<td>$2.71</td>
<td>$18.00</td>
</tr>
<tr>
<td>August-14</td>
<td>$22.85</td>
<td>$3.75</td>
<td>$2.56</td>
<td>$16.54</td>
</tr>
<tr>
<td>September-14</td>
<td>$22.28</td>
<td>$3.65</td>
<td>$2.38</td>
<td>$15.25</td>
</tr>
<tr>
<td>October-14</td>
<td>$21.87</td>
<td>$3.63</td>
<td>$2.27</td>
<td>$15.07</td>
</tr>
<tr>
<td>November-14</td>
<td>$21.45</td>
<td>$3.63</td>
<td>$2.26</td>
<td>$15.66</td>
</tr>
</tbody>
</table>

Data above is from last trade for 07-25-2014
# LGM-Dairy: How Did It Perform?

<table>
<thead>
<tr>
<th>Insurance Year</th>
<th>Premiums Paid</th>
<th>Indemnities Received</th>
<th>Loss Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>2008/2009</td>
<td>287,201</td>
<td>718,035</td>
<td>2.50</td>
</tr>
<tr>
<td>2009/2010</td>
<td>781,589</td>
<td>280,566</td>
<td>0.36</td>
</tr>
<tr>
<td>2010/2011</td>
<td>25,012,757</td>
<td>64,738</td>
<td>&lt;0.01</td>
</tr>
<tr>
<td>2011/2012</td>
<td>19,153,150</td>
<td>1,317,954</td>
<td>0.07</td>
</tr>
<tr>
<td>2012/2013</td>
<td>16,878,326</td>
<td>2,634,190</td>
<td>0.16</td>
</tr>
<tr>
<td>2013/2014</td>
<td>10,902,308</td>
<td>0</td>
<td>0.00</td>
</tr>
</tbody>
</table>
Research Questions

- Are dairy and feed (corn and soybean meal) futures prices unbiased?

- Are implied risk measures inferred from dairy and feed options unbiased prediction of forward volatility?
Data Issues

- Need to predict margins at long-horizons (6-24 months)

- Dairy derivatives data only available since 1998
LGM-Dairy Assumptions

\[ f_t = E_t( p_T ) \]

1. Futures prices are unbiased predictors of terminal (futures) prices.

\[ \psi_t ( \ln p_T ; f_t, \sigma_t ) \sim N \left( \ln f_t - \frac{1}{2} \sigma_t \tau, \sigma_t^2 \tau \right) \]

2. Terminal futures prices are lognormally distributed.
3. The log-price conditional variance is equal to CRR implied volatility squared times time-to-maturity.

4. Milk and feed prices are uncorrelated.
LGM-Dairy Assumptions

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Percentage Prediction Error (PPE)

\[ f_{t,i} = E_t\left(p_{T,i}\right) \]

\[ E_t\left(\frac{f_{t,i} - p_{T,i}}{f_{t,i}}\right) = 0 \]

\[ PPE_{t,i} \equiv \frac{f_{t,i} - p_{T,i}}{f_{t,i}} \times 100 \]

\[ \overline{PPE}_i = \frac{1}{N} \sum_{l=1}^{N} \frac{f_{l,i} - p_{l,i}}{f_{l,i}} \times 100 \]

\[ = \frac{1}{N} \sum_{l=1}^{N} PPE_{l,i} = 0 \]
Testing for Biases in Option Premiums

\[ \psi_t (\ln p_T; f_t, \sigma_t) \sim N \left( \ln f_t - \frac{1}{2} \sigma_t \tau, \sigma_t^2 \tau \right) \]

\[ E_t \left( \sigma_{t,i}^2 \tau - \left( \ln p_{T,i} - \left( \ln f_{t,i} - \frac{1}{2} \sigma_{t,i}^2 \tau \right) \right)^2 \right) = 0 \]

\[ E_t \left( \ln p_{T,i} - \left( \ln f_{t,i} - \frac{1}{2} \sigma_{t,i}^2 \tau \right) \right)^2 = \sigma_{t,i}^2 \tau \]

\[ E_t \left[ \frac{\ln p_{T,i} - \left( \ln f_{t,i} - \frac{1}{2} \sigma_{t,i}^2 \tau \right)}{\sigma_{t,i} \sqrt{\tau}} \right]^2 = 1 \]
Root Mean Square Standardized Prediction Error

\[
E_t \left[ \ln p_{T,i} - \left( \ln f_{t,i} - \frac{1}{2} \sigma_{t,i}^2 \tau \right) \right]^2 = 1
\]

\[
RMSSP E_{i} = \sqrt{\frac{1}{N} \sum_{l=1}^{N} \left[ \ln p_{l,i} - \left( \ln f_{l,i} - \frac{1}{2} \sigma_{l,i}^2 \tau \right) \right]^2} = 1
\]
Root Mean Square Standardized Prediction Errors

- **LGM-Dairy** Insurable Month
- **Milk** Soybean Meal Corn

![Graph showing Root Mean Square Standardized Prediction Errors for LGM-Dairy Insurable Month with data points for Milk, Soybean Meal, and Corn.](image-url)
Are PPE and RMSSPE consistent with null hypothesis?

1. Estimate parameters of data generating process, imposing restrictions consistent with null.

2. Generate K data samples.

3. For each bootstrapped sample, calculate PPE and RMSSPE.

4. Distribution of bootstrapped PPEs (RMSSPE) will be centered at zero. For $\alpha$ confidence level, choose PPE at $\alpha/2$ and $1-\alpha/2$ quantiles of PPE distribution.

5. If original PPE (RMSSPE) falls outside the bootstrapped confidence interval, reject the null hypothesis.
Simulating terminal prices:

\[ p_{T,i}^* = \exp\left( z_{T,i}^* \times \sigma_{t,i} \sqrt{\tau} + \left( \ln f_{t,i} - 0.5 \times \sigma_{t,i}^2 \tau \right) \right) \]

Dealing with overlapping simulation horizons:

\[ z_t = \sum_{m=1}^{p} \phi_m z_{t-m} + \sum_{m=1}^{q} \alpha_m \varepsilon_{t-m} + \varepsilon_t \quad \varepsilon_t ~ N\left(0, \sigma_\varepsilon^2\right) \]

How should we estimate ARMA(p,q) parameters so that they:
1) Are inferred from observed data?
2) Obey H1 (unbiasedness of futures price)?
3) Obey H2 (unbiasedness of implied volatilities)?
Overlapping horizons: consistency with no bias in futures

\[ z_t = \sum_{m=1}^{p} \phi_m z_{t-m} + \sum_{m=1}^{q} \alpha_m \varepsilon_{t-m} + \varepsilon_t \quad \varepsilon_t \sim N\left(0, \sigma^2_\varepsilon\right) \]

1. Unbiasedness \(\rightarrow\) Efficiency
2. Efficiency implies ARMA \((p,q)\) must be truncated to MA\((j-1)\), where \(j\) is the highest futures nearby index used for \(i\)-th insurable month.

\[ z_t = \sum_{m=1}^{q} \alpha_m \varepsilon_{t-m} + \varepsilon_t \quad \varepsilon_t \sim N\left(0, \sigma^2_\varepsilon\right) \]
Overlapping horizons: consistency with no bias in options

\[ z_t = \sum_{m=1}^{q} \alpha_m \varepsilon_{t-m} + \varepsilon_t \quad \varepsilon_t \sim N(0, \sigma^2_\varepsilon) \]

\[ \text{Var}_t(\ln p_T) = \sigma^2_{t,T} \tau \]

\[ p^*_{T,i} = \exp \left( z^*_{T,i} \times \sigma_{t,i} \sqrt{\tau} + \left( \ln f_{t,i} - 0.5 \times \sigma^2_{t,i} \tau \right) \right) \]

\[ \text{Var}_t(\ln p^*_{T,i}) = \sigma^2_{t,i} \tau \times \text{Var}_t(z^*_{T,i}) \]

\[ \text{Var}_t(z^*_{T,i}) = 1 \]

\[ \sigma^2_\varepsilon = \frac{1}{1 + \sum_{m=1}^{j-1} \alpha^2_m} \]

\[ Z \sim N(0, \sigma^2_\varepsilon) \]
Estimating MA coefficients

\[ z_t = \sum_{m=1}^{j-1} \alpha_m \varepsilon_{t-m} + \varepsilon_t \quad \varepsilon_t \sim N\left(0, \sigma^2_\varepsilon\right) \]

\[ u_{t,i} = \Psi_t \left( \ln p_{T,i}; \ln f_{t,i} - \frac{1}{2} \sigma^2_{t,i} \tau, \sigma^2_{t,i} \tau \right) \]

\[ \tilde{z}_{t,i} = \Phi^{-1} \left( u_{t,i} \right) \]

\[ z_{t,i} = \frac{\tilde{z}_{t,i} - \tilde{\mu}_i}{\tilde{\sigma}_i} \]

\[ z_{T,i}^* = \sum_{m=1}^{j-1} \hat{\alpha}_m \varepsilon_{t-m} + \varepsilon_t \quad \varepsilon_t \sim N \left(0, \frac{1}{1 + \sum_{m=1}^{j-1} \alpha^2_m} \right) \]
Test for Bias in Futures Prices: Class III Milk

Prediction Error (%) vs. Insurable Month
Test for Bias in Futures Prices: Corn

Prediction Error (%) vs. Insurable Month

Prediction error ranges from -15% to 15%.
Test for Bias in Futures Prices: Soybean Meal

Prediction Error (%) vs. LGM-Dairy Insurable Month
Test for Bias in Implied Volatilities: Class III Milk
Test for Bias in Implied Volatilities: Corn

Root Mean Square Standardized Prediction Error

Insurable Month

1 2 3 4 5 6 7 8 9 10
Test for Bias in Implied Volatilities: Soybean Meal

Soybean Meal

Index

Root Mean Square

Standardized Prediction Error

LGM-Dairy Insurable Month
Conclusions

1. Class III Milk and Corn markets show no sign of biases in futures prices. Soybean meal futures downward biased.

2. Tests for implied volatility biases
   • Corn options on futures accurately reflect risk in the corn futures market.
   • For soybean meal, option unbiasedness only holds if we assume that SBM futures prices embody risk premium.
   • Class III Milk options seem to be underpriced. Further analysis with long-straddle based trading programs indicate profitable trading strategies even after accounting for transaction risks.

3. None of these findings help explain low LGM-Dairy loss-ratios. If anything, correcting for these biases would increase LGM-Dairy premiums.
Parametric Bootstrap Tests for Futures Price and Implied Volatility Biases With Application to Rating Dairy Margin Insurance

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